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CS 215

Homework 5

CH 4.1

1. Basic Step   
   P(1): 1 \* 1! = (1 + 1)! – 1  
    1\* 1 = (2)! – 1  
    1 = 2 – 1  
    1 = 1  
     
   Inductive Step:   
   P(k) = 1 \* 1! + 2 \* 2! + … + k \* k! = (k + 1)! - 1  
   P(k + 1) = 1 \* 1! + 2 \* 2! + … + k \* k! + (k +1) \* (k + 1)! = ((k + 1) + 1)! – 1 = (k + 2)! - 1  
     
   (k + 1)! – 1 + (k + 2)! – 1 = (k + 1)! + (k + 2)! – 2

CH 5.1

1. A bit string of length eight has eight slots for which there are two possible choices for each slot. Thus the total possible number of bit strings would be 2 \* 2 \* 2 \* 2 \* 2 \* 2 \* 2 \* 2 or 28.
2. The first and last digits must be one. Thus, the total number of possible bit strings would be 2n-2.
   1. Repetition is not allowed, so as each slot is filled, that number is eliminated from the pool. Thus the total possible combinations would be 10 \* 9 \* 8 \* 7 = 5040.
   2. The first three slots have 10 possible options. The last slot, however, must be even. Thus, the last slot has only 5 options. Thus, the total number of possibilities would be   
      10 \* 10 \* 10 \* 5 = 103 \* 5 = 5000.
   3. Only one digit will have 10 choices, thus the total possible combinations would be   
      10 \* 1 \* 1 \* 1 = 10. The non-nine number could be in any of the four positions, so it is 10 \* 4 = 40.

CH 5.3

* 1. Three slots must be filled by a one. The rest of the slots could be either zero or one. Each slot is unique as a number one in one slot is different from a number one in another slot. Thus, there are C(12, 3) bit strings of length 12 that contain exactly 3 ones.
  2. Any given bit string in this scenario could contain 1, 2, or three ones. Thus, the total number of possibilities is C(12, 1) + C(12, 2) + C(12, 3).
  3. In this case, a given bit string could contain 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12 ones. Thus, the total number of possibilities is C(12,3) + C(12,4) + C(12,5) + C(12,6) + C(12,7) + C(12,8) + C(12,9) + C(12,10) + C(12,11) + C(12,12)
  4. A bit string cannot have any null values. Also, there are only two values for a given bit. Thus, for a bit string of length 12 to have an equal number of ones and zeros, there must be six of each. The total number of possibilities for a bit string that has six ones and six zeros is C(12, 6) + C(6, 6). The second part is superfluous and just shows that the remaining six slots will be filled by the opposite bit.
  5. The string ED must occur as a block. This creates 7 objects: the string ED and the remaining individual letters. Thus the number of permutations is 7!
  6. Here there is a string of length three. This creates six distinct objects for a valid string. Thus the number of permutations is 6!
  7. Here there are 5 unique objects: BA, FGH, C, D, and E. Thus, the number of permutations is 5!
  8. There are 5 unique objects here as well, so the number of permutations is again 5!
  9. The committee must consist of five members out of the 16 available. There must be at least one woman, but it is possible for all seven women to be picked. Thus, the number of ways to form a committee is C(7, 1) \* C(15, 4) because after choosing the required one woman, there are 15 faculty members and 4 slots left.
  10. Here, there must be at least one woman and one man. Thus, the number of ways to form a committee here is C(7, 1) \* C(9, 1) \* C(14, 3) because after choosing one man and one woman, there are 14 faculty members and three slots left.

CH 5.4

1. (x + y)5 (x + y)n = Σ n 🡪 j=0 (n j) xn – jyj= (n 0)xn + (n 1)xn-1y + (n 2)xn-2y2 + … + (n n-1)xyn-1 + (n n)yn, thus:  
   x5 + x5-1y5-4 + x5-2y5-3 + x5-3y5-2 + x5-4y5-1 + y5The coefficients of the terms can be found by as a combination of C(5,0), C(5,1), C(5,2)…C(5,5) thus:  
   x5 + 5x5-1y5-4 +10 x5-2y5-3 +10 x5-3y5-2 + 5x5-4y5-1 + y5
2. Since the exponent of x is 5, the Binomial Theorem shows that it must be the 9th term in the expansion. Thus, the coefficient would be C(13, 8) as the value of r in the combination formulas start at zero.